Calculus 140, section 4.1 Maximum and Minumum Values

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Definition 4.1: "a. A function f **has a maximum value on a set *I*** if there is a number d in I such that $f(x) \le f(d)$ for all x in I…We call $f(d)$ the **maximum value of *f* on *I***.

b. A function f **has a minimum value on a set *I*** if there is a number c in I such that $f(x) \ge f(c)$ for all x in I…We call $f(c)$ the **minimum value of *f* on *I***.

c. A value of f that is either a maximum value or a minimum value on I is called an **extreme value of *f* on *I***."

In non-technical terms, a maximum will be a high point on a graph (top of a hill) and a minimum will be a low point on a graph (bottom of a valley).

The text illustrates situations for which a function does not have a maximum value or a minimum value or possibly neither: the domain is an open interval, the function is not continuous on an interval, the domain is unbounded.

Theorem 4.2 [Maximum-Minimum Theorem]: "Let f be continuous on a closed, bounded interval $[a, b]$. Then f has a maximum and a minimum value on $[a, b]$."

Theorem 4.2 is proven in the Appendix of your text.

Once we know we have a maximum and/or a minimum, how do we locate it?

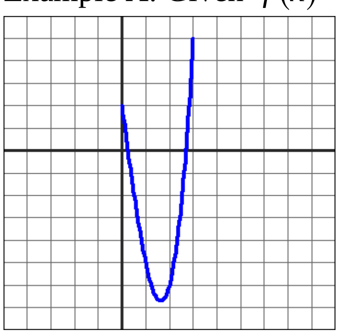
Theorem 4.3: "Suppose c is an interior point of an interval I, and $f(c)$ is an extreme value of f on I. If $f'(c)$ exists, then $f'(c) = 0$."

In the text there is a "proof by contradiction".

The text also notes that Theorem 4.3 implies that, if an extreme value of f occurs at an interior point of an interval I, then either $f'(c) = 0$ or f is not differentiable at c. This gives us a method for determining the maximum and minimum values of a function on a closed interval $[a, b]$.

1. Find $f'(x)$ and determine critical values, i.e. values where either $f'(c) = 0$ or f is not differentiable.
2. Evaluate f(each critical value), $f(a)$ and $f(b)$ to identify maximum and minimum values.

Example A: Given $f(x) = x^3 - 8x + 2$, find the maximum and minimum values on the interval [0, 3].



first derivative:

critical values:

critical points:

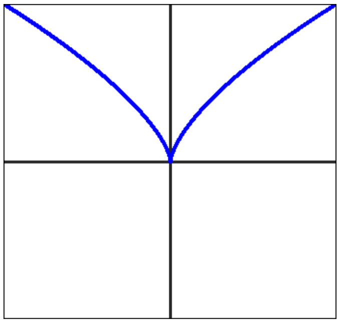
function evaluated at left endpoint:

function evaluated at right endpoint:

maximum value and location:

minimum value and location:

Example B: Given the function $f(x) = x^{2/3}$, find the maximum and minimum values on the interval [–1, 1].



first derivative:

critical values:

critical points:

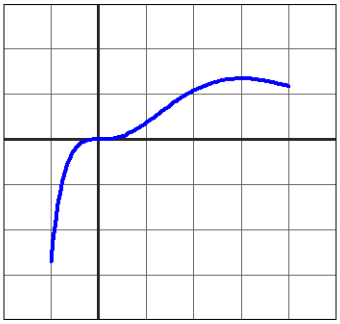
function evaluated at left endpoint:

function evaluated at right endpoint:

maximum value and location:

minimum value and location:

Example C: Given the function $f(x) = \frac{x^3}{e^x}$, find the maximum and minimum values on the interval [–1, 4].



first derivative:

critical values:

critical points:

function evaluated at left endpoint:

function evaluated at right endpoint:

maximum value and location:

minimum value and location:

special note about the point (0, 0):

Example D: Public health officials use rates of change to quantify the spread of an epidemic into an equation, which they then use to determine the most effective measures to counter it. A recent measles epidemic followed the equation $y = 45t^2 - t^3$ where y = the number of people infected and t = time in days.

a) What is the domain of this function?

b) What are the maximum and minimum number of people infected in the course of the epidemic?

Example E: Perhaps you have already encountered versions of the infamous corral problem favored by Math 115 and 140 course coordinators. A farmer has 900 feet of fencing with which to build a pen for his animals, and being a frugal sort doesn't want to buy any more fencing. He needs two pens, but can build them adjacent to each other, sharing one side as in the diagram to the left. Find the dimensions that will give the maximum area.